Kaktovik Numerals

A different number system

Willi Egger

We are used to counting in the Arabic number system, which is base 10. However, we can use a number system with a different base. The native Alaskan nations, like Iñupiat[1], Inuit, and Yupik, for example, use base 20 to count. This article discusses the Iñupiaq number system.

1. Introduction

In April 2023 Hraban Ramm posted a link to the number system used by the Alaskan Iñupiat. It's a base 20 number system, and there is a remarkable story behind it.

Only a couple of hours after this post, Hans created and posted a Type-3 font for typesetting these numbers in ConTEXt.

It was for me an intriguing story, and I started reading about this number system and trying it out in ConTEXt.

2. A glance at number systems

There are various number systems in the world. Some of them were only used in the distant past. This table shows some of these systems. First we must differentiate between so called positional systems and non-positional systems.

Positional notation

System	Base	Digits	Comment
Arabic	10	10	0 to 9, Europe, Middle East and Asia
Binary	2	2	0 and 1
Octal	8	8	0 to 7
Hexadecimal	16	16	0 to 9 and A to F
Aztec	20	4	dot, flag, "fir" tree, incense bag
Maya	20	3	"zero", dot, line
Inuit	20	3	zero-sign, "vertical" and "horizontal" lines

[1] Iñupiat is the name of the people, Iñupiaq their language, but usage seems to differ. (the ed.)

Non-positional notation

System	Base	Symbols	Comment
Roman numerals		7	I,V,X,L,C,D,M

2.1 Traces of base 20 number systems in Europe

Base 20 number systems were used in the Aztec and Maya cultures, and Inuit still use this system. Even in Europe we still can find traces of base 20 number systems. You might know how French count 80 and 90, although they use the Arabic notation:

French 80 → quatre-vingt
90 → quatre-vingt-dix

On the other hand the French speaking people in Switzerland use:

Swiss 80 → ottante
90 → nonante

2.2 The traditional Iñupiaq number system

The way traditional numbers are given names in the Iñupiaq language is more complicated than in other languages. This table shows that many of them are composites, with either a subtraction or an addition. This list is from Wikipedia [3].

Iñupiaq	written as	Arabic
atausiq		1
malġuk		2
piŋasut		3
sisamat		4
tallimat		5
itchaksrat		6
tallimat malġuk	five+two	7
tallimat piŋasu	five+three	8
quliŋŋuġutaiaq	ten-one	9
qulit		10
qulit atausiq	ten+one	11
qulit malġuk	ten+two	12
qulit piŋasut	ten+three	13
akimiaġutaiaq	fifteen-one	14
akimiaq		15
akimiaq atausiq	fifteen+one	16
akimiaq malġuk	fifteen+two	17
akimiaq piŋasut	fifteen+three	18
iñuiññaq		20
iñuiññaġutaiaq	twenty-one	21
iagiññaq		400

2.3 A problem with the traditional number system

The base 20 number system traditionally used by the Iñupiat is very different to the Arabic number system. It is difficult for someone used to the Iñupiaq system to communicate with someone used to the Arabic base 10 number system. Today, many things are described with numbers. It was found that Iñupiat students were scoring significantly lower than other students of the same age who used Arabic numbers.

2.4 How this problem was solved

In the 1990s, a group of Iñupiat high-school students in the most northern part of Alaska devised a new number system. In some ways it is a visual representation of the traditional system.

They formulated five goals to be met for the new number system.

- Simplicity: symbols must be easy to memorise.
- Iconicity: symbols need to be like pictures, with a clear relation between the symbols and their meanings; like emojis.
- Symbols must be easy to write by hand without interrupting the flow of writing.
- The symbols must be easily distinguished from other symbols of counting systems (such as Arabic numbers).
- The symbols must be attractive to look at.

The name of the new system, Kaktovik (Iñupiat: Qaaktuġvik), refers to the town where the students went to school.

3. The Kaktovik digits

Like the traditional Iñupiaq number systems, the Kaktovik system is base 20. A base 20 number system requires 20 digits. The Kaktovik system is brilliantly simple. Only two elements (a vertical and a horizontal stroke) are used to represent the digits, plus an additional symbol for zero, which does not exist in Iñupiaq.

The Kaktovik system supports the traditional Iñupiaq way of counting. For example, in both systems, the digit 7 is 5 + 2. When you write the numbers quickly by hand, you can slope them like italics.

Number 1 is a vertical stroke. Successive digits are written by adding an additional vertical stroke to the previous digit. Every five digits the vertical strokes are removed and a horizontal stroke added. Because the Iñupiaq language does not have a word for zero, this is an additional sign. It looks like crossed arms or an x, indicating “nothing” or “in this place there is no calculation”.

The complete set of 20 digits is:

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19

An easier way of showing the the system is a table with 5 columns and 4 rows:

0	1	2	3	4
5	6	7	8	9
10	11	12	13	14
15	16	17	18	19

4. Exploring the Kaktovik number system

When writing numbers bigger than 19 we must use more than one place. And remember that the Kaktovik number system is positional, like the Arabic number system. These tables show the main differences between the two number systems:

Arabic				
10^4	10^3	10^2	10^1	10^0
10000	1000	100	10	1

Kaktovik				
20^4	20^3	20^2	20^1	20^0
160000	8000	400	20	1

When counting up in the Arabic system, we use the second place when reaching 10 (10^1), the third place when reaching 100 (10^2), the fourth place when reaching 1000 (10^3), and then 10^4 will use place 5. And so on.

In contrast, the Kaktovik system uses the second place once 20 (20^1) is reached. When reaching 400 (20^2) the third place is used. Only when reaching 8000 (20^3) do we need the next place. Using the fifth place we can count to 160000.

Writing large numbers with base 20 needs fewer places than with base 10.

Arabic	Iñupiaqcomposition	Kaktovik
20	$1 \times 20^1 + 0 \times 20^0$	
30	$1 \times 20^1 + 10 \times 20^0$	
40	$2 \times 20^1 + 0 \times 20^0$	

100	$5 \times 20^1 + 0 \times 20^0$	
1000	$2 \times 20^2 + 10 \times 20^1 + 0 \times 20^0$	
2023	$5 \times 20^2 + 1 \times 20^1 + 3 \times 20^0$	

These examples used integers. The Kaktovik system also works perfectly for numbers with decimal places. Examples hereof can be found on Wikipedia [2].

5. Examples of calculations

The Kaktovik system is suitable for calculations. Here follow some simple examples. More in-depth calculations are shown on Wikipedia [2].

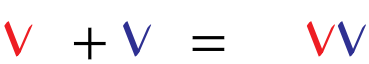
An important aspect of the Kaktovik number system is the fact that the numbers have patterns of vertical and horizontal strokes. Calculations can therefore be performed by matching patterns. The examples shown here are derived from a YouTube film created by “The Ferret” [6].

Example 1: Simple addition

Arabic: $2 + 2 = 4$

No relation can be detected between the number 2 and the answer, 4.

Kaktovik:

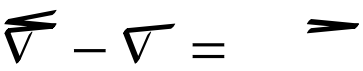


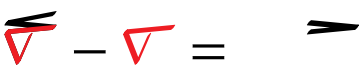
Looking at the patterns, we see that the answer is just the patterns of the left side of the equation added together. This shows that one can do calculations even without knowing the value of the numbers.

Example 2: Simple subtraction

Arabic: $17 - 7 = 10$

Kaktovik:





In both numbers we find a pattern. Remove strokes from the first number that are in the second number. Then rotate the result by 180 degrees to represent the normal writing direction. Again, we found the result simply by looking at the patterns.

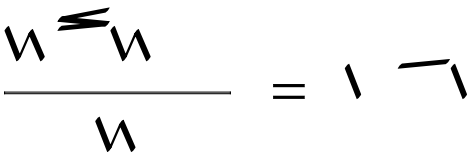
Example 3: Division with no remainder

This is a simple division.

A reminder: In a division, the numerator is on top and the denominator on the bottom. And, in the number 321, 3 is in the third place, 2 is in the second place and 1 is in the first place.

Arabic: $\frac{1503}{3} = 501$

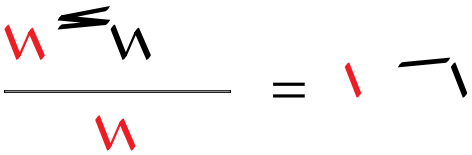
Kaktovik:



We could calculate the result in the Arabic number system and write it as Kaktovik numbers. However we can also solve this example by matching patterns.

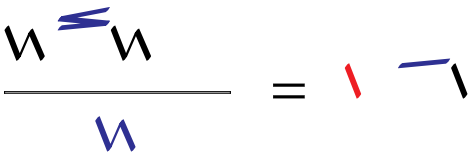
First step:

Compare the number in the third place of the numerator with the number in the denominator. The pattern is identical. Write the first digit of the result, a vertical stroke.



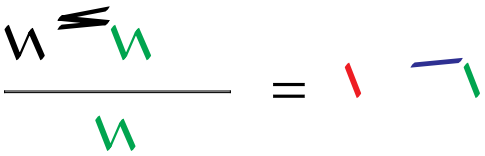
Second step:

Compare the digit in the second place of the numerator with the denominator. The patterns look similar, except that the numerator is rotated by 90°. Write the second digit of the result, a horizontal stroke.



Third step:

Compare the digit in the first place of the numerator with the denominator. The patterns are the same. Write the third digit of the result, a vertical stroke.

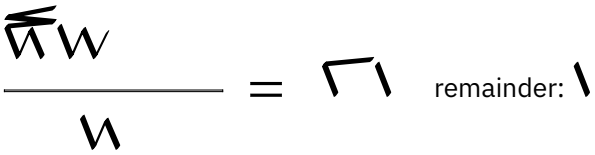


Again, the equation can be solved just by matching patterns.

Example 4: Division with a remainder

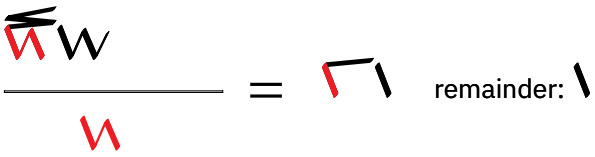
Arabic: $\frac{364}{3} = 121$ remainder 1

Kaktovik:



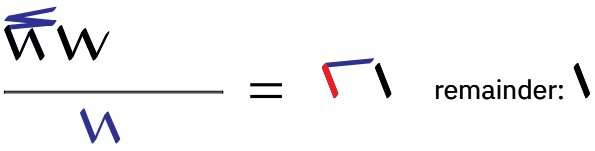
First step:

We see that the pattern in the denominator is contained in the second place of the numerator. Write the first digit of the result, a vertical stroke.



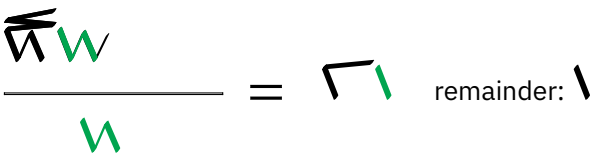
Second step:

The pattern of the denominator is contained in the top part of the second place of the numerator, but it is rotated 90°. Write a horizontal stroke on top of the existing first digit of the result.



Third step:

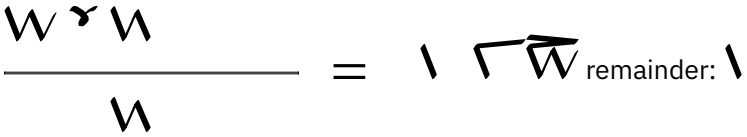
The pattern of the denominator is contained in the first place of the numerator. Write the second digit of the result, a vertical stroke. Removing the pattern of the denominator from the first place of the numerator leaves a single vertical stroke, which is the remainder of the division.



Example 5: Limits of Kaktovik

Arabic: $\frac{1603}{3} = 534$ remainder 1

Kaktovik:



We can not solve this using the methods of the above examples. Wikipedia [2] has more examples of calculating with Kaktovik numbers.

6. Further information

For a deeper exploration and understanding the number system, visit Wikipedia [2]. In order to see the Kaktovik numbers it is necessary to install a font with these numbers. There are the fonts SIL GentiumKaktovik [7] or Noto Sans Symbols 2 [8], which are free of charge.

7. Hans's Type-3 font

A couple of hours after start of the conversation about the Kaktovik numbers, Hans came up with a MetaFun module, which generates a PostScript Type-3 font. This font can be used to experiment in ConTEXt with this number system.

In order to enable cooperation between your body font and the Kaktovik number font, set up the Kaktovik font before you set up the body font. This is because the Kaktovik font is invoked through a font feature, which is added to "default".

The following code does the magic:

```
\useMPlibrary[kaktovik]

\definefontfeature
 [default]
 [default]
 [metapost=kaktovik]
```

The advantage of having this Type-3 font is that the Kaktovik number are glyphs, which makes it possible to cut and paste them.

8. Conclusion

The Iñupiat students developed a new number system that is based on the traditional Iñupiaq number system. The new system can be used for calculations in two ways: using methods used in Arabic number calculations or by matching patterns.

Once the Kaktovik system was adopted, the results of math exams by Iñupiat students improved steadily. Some even surpassed the marks of non-indigenous students using Arabic numbers.

The success of the Kaktovik number system relies on the brilliant and simple way it represents numbers visually.

9. Links and further information on the Kaktovik number system

[1] *en.wikipedia.org/wiki/List_of_numeral_systems*
[2] *en.wikipedia.org/wiki/Kaktovik_numerals*
[3] *en.wikipedia.org/wiki/I%C3%B1upiaq_numerals*
[4] *www.youtube.com/watch?v=EyS6FfczH0Q&ab_channel=Artifexian*
[5] *www.youtube.com/watch?v=fIZB4bRwxqI&ab_channel=Dave%27sMathVideos*
[6] *www.youtube.com/watch?v=ObRFHiU_r9I&ab_channel=TheFerret*
[7] SIL GentiumKaktovik from *languagetools-153419.appspot.com/ik/downloads/*
[8] Noto Sans Symbols 2 from *fonts.google.com/noto/specimen/Noto+Sans+Symbols+2*

10. Thanks

- Thank you Hraban, for sharing the information about the Kaktovik number system on the mailing-list.
- Thank you Hans, for the Type-3 font, which allows the use of Kaktovik numbers in ConTEXt and enabled this article to be written.